Is Talk Cheap Online:
Strategic Interaction in A Stock Trading Chat Room

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Abstract:

We consider a model of an internet chat room with free entry but secure identity. Traders exchange messages in real time of both a fundamental and non-fundamental nature. We explore conditions under which traders post truthful information and make trading decisions. We also describe an equilibrium in which momentum and hybrid traders profit from their exposure to informed traders in the chat room. The model generates a number of empirical predictions:(1) traders with middle skill level communicate most often; (2) All but the most informed traders learn from public information about prices, and they optimally follow informed traders; (3) Traders follow informed traders more often. We test and affirm all three predictions using a unique data set of chat room logs from the Activetrader Financial Chat Room.

Keywords: chat room; strategic information; individual traders; behavioral finance;

JEL Codes: G14;

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1. Introduction

Wall Street has a lot in common with Madison Avenue. There is a great deal of information disseminated to influence portfolio selection. There are numerous communications among professionals and here comes out the questions: will trader A take positions in a stock after trader B says he has loaded up? When will trader B tell the truth and when will he lie? There is no effective way to study the real time effects of such informal communications among professional investors. However, when stock trading chat rooms come out, we now can study similar interactions among individual semi-professional traders. This paper studies the influence of communications among individual day traders on their trading decisions. And we takes advantage of a unique data set of the chat room posts of more than 1,000 individual day traders and studied their interaction and transactions in time series.

There are two advantages that individual day traders are good objectives to study the effect of informal communications in trading decision making: (1) unlike professionals, they do not have any trading rules or trading guidelines forced on them, which make their trades more personal-decision driven; (2) unlike professionals, they do not have enough capital to verify others' news/rumors/ideas by testing market liquidity, which makes the influence of the real-time interaction on their trading decisions more easily to study.

There is now an established literature on the performance of individual traders. Odean (1999) initiated the studies on individual traders and documented poor returns in a sample of more than 35,000 households. He attributes the underperformance to both overtrading and the disposition effect, the tendency to sell winners and hold losers.

Some recent papers, including Coval, Hirshleifer, and Shumway (2005) and Niccolosi, Peng, and Zhu (2003), have suggested that traders might gain experience that improves their performance over time. Mizrach and Weerts (2007) show that skills may be stock specific. As far as we know, the literature has not looked at the real-time interactions between individual traders, perhaps because of data limitations.

We model individual day traders' interactions as a dynamic game and study several basic questions: Who communicates the most? When do they communicate? And why? The model establishes three strong empirical predictions: (1) Neither the most informed nor the most uninformed traders communicate most often; (2) All but the most informed traders learn from public information about prices, and optimally follow more informed traders; (3) Traders follow the most informed traders, instead of the most active ones, more often.

We typically don't observe the message traffic between traders and their brokers. And we also don't see trading decisions linked directly to their posts. Antweiler and Frank (2004) study Internet bulletin board posts, but these are not observed in real time.

This paper takes advantage of a unique data set of the chat room posts of more than 1,000 individual traders, with which we confirm the three main empirical predictions of our model.

The paper is organized as follows: Section 2 describes the equilibrium if traders cannot communicate; Section 3 describes the equilibrium with an informal communication group and the empirical implications; Section 4 introduces the data; Section 5 presents our empirical results; Section 6 concludes and speculates about the generalizability of the results.

2. Model

Our model describes how traders with different information levels trade in the market and how they move the stock price according to their expectations.

There is a risky asset V with initial value v_0. Information is released at time $t = \Gamma$ which changes the risky asset's value to $\widetilde{v}$. The value of $\widetilde{v}$ depends on the state of the world, which takes three values from the set $\widetilde{\omega} = \Omega = \left\{\omega^-, \omega^0, \omega^+\right\}$. $\widetilde{v} = v_0 + \widehat{v}$ in state ω^+, $\widetilde{v} = v_0$ in state ω^0 and $\widetilde{v} = v_0 - \widehat{v}$ in state ω^-. The prior probability of each state $\left\{\omega^+, \omega^0, \omega^-\right\}$ is $\{p, 1-2p, p\}$.

We divide $[0, \Gamma]$ into 3 periods and $\widetilde{v}$ is revealed as information is released at period 2. The time-discount factor is denoted as β.

There are four kinds of traders in the market: informed traders S_I, hybrid traders S_H, momentum traders S_M and noise traders. Noise traders trade for liquidity reasons and simply add noise into price. We study three kinds of traders' optimal strategy: informed traders S_I, hybrid traders S_H and momentum traders S_M, who receive signals and trade for profits according to all information they can get, and we also take into account the noise term in the price which comes from noise traders' behaviors.

Each trader i, except noise traders, receives a signal $\theta_i \in \Theta = \left\{\theta^1, \theta^2, \theta^3, \theta^4, \theta^5, \theta^6\right\}$ at period 0, where $\theta^1 = \{0, +\}$, $\theta^2 = \{0, -\}$, $\theta^3 = \{+\}$, $\theta^4 = \{-\}$, $\theta^5 = \{0\}$, $\theta^6 = \{+, 0, -\}$. Signal $+$ indicates state ω^+, signal $-$ indicates state ω^-, and signal 0 indicates state ω^0.

Informed traders S_I are the most skillful traders, capable of estimating the true value of the asset precisely before information is released in period 2. In the model, they have perfect information in their signal. S_I receive signal $\theta^3 = \{+\}$ in state ω^+, $\theta^5 = \{0\}$ in state ω^0 and $\theta^4 = \{-\}$ in state ω^-.

Momentum traders S_M are the least skillful traders and their signals have no information. They receive signal $\theta^6 = \{+, 0, -\}$ in all the three states.

Hybrid traders S_H's skills are between informed traders S_I's and momentum traders S_M's. They have some capability to estimate the price but not so good as informed traders. They have imperfect information in their signal. S_H receive signal $\theta^1 = \{0, +\}$ in state ω^+ , signal $\theta^2 = \{0, -\}$ in state ω^-; And in state ω^0, S_H receive signal $\theta^1 = \{0, +\}$ with probability $\frac{1}{2}$ and $\theta^2 = \{0, -\}$ with probability $\frac{1}{2}$. Thus, from signal $\theta^1 = \{0, +\}$, S_H can deduce that it is state ω^+ with probability $2p$ and state ω^0 with probability $1 - 2p$. Similarly, from signal $\theta^2 = \{0, -\}$, S_H can deduce that it is state ω^- with probability $2p$ and state ω^0 with probability $1 - 2p$.

Noise traders trade for liquidity reasons and their trades are random. Since they do not trade on any information, we assume they receive no signal.

[Insert Table 1 Here]

A trader's type and signal are private information to her. Suppose the number of traders S_I, S_H and S_M in the market are QQ_I, QQ_H and QQ_M.

We assume each trader can only hold 1 unit (long position), -1 unit (short position) and 0 unit of risky asset. The changes of their positions generate the order flow. All orders from S_I, S_H and S_M are limit orders with their expectation of asset value as limit price.

At period s, trader i's action is denoted as $a_s^i \in \Lambda_1 = \{-1, 0, 1\}$, where $\{-1, 0, 1\}$ is the action set, 1 means planning to hold 1 unit (long position) of risky asset, S means planning to hold -1 unit (short position), and 0 means planning to hold 0 unit. Assume S_I, S_H and S_M all hold position 0 at the beginning, period 0. Trader i's strategy at period $s = 0, 1$ is denoted as $a^i = \left\{a_0^i, a_1^i\right\}$, where a_0^i is trader i's intended position at period 1 and a_1^i is trader i's intended position at period 2. Trader i's trade at period 0 is the change of her positions from period 0 to period 1, and her trade at period 1 is the change of her position from period 1 to period 2. For example, $\left\{a_0^i = 1, a_1^i = 0\right\}$ means trader i plans to hold position $+1$ at period 1 and position 0 at period 2. And so, at period 0, she submits an order to buy 1 unit of asset in order to hold $+1$ position at period 1. If her order is executed at period 1, she submits an order to sell her long position in order to hold 0 position at period 2. If her order is not executed at period 1, she does not submit any order and maintain 0 position at period 2. As we discussed, the change of trader i's positions shows her order flow.

At each period, all orders are submitted to market maker and will influence the price at next period. The aggregate order flow at period $t-1$ has impact on the asset's price at period t. Assume each unit of order flow has the same market impact λ on price.

The price at period t, denoted as P_t, is decided by the market aggregate expectation of risky asset's value at period $t-1$, plus noise ε_t coming from noise traders' liquidity traders, where $\varepsilon_t \sim N(0, \sigma_\varepsilon^2)$. Specifically, P_t equals to the aggregate demand/supply plus the realizations of noise, where the aggregate demand/supply at period t equals to the aggregate demand/supply at period $t-1$ plus the market impact from the overall order flows submitted at period $t-1$.

Let us summarize the process: At period 0, traders receive their signals and submit their orders based on their signals. At period 1, traders observe the price, which is influenced by the aggregate order flow at period 0, and submit their orders according to all private and public information they can attain. At period 2, information is released and traders form the homogenous expectation of the asset's value, the realization of $\widetilde{v}$. Thus, they submit limit orders with the true value as limit price and push the price,excluding the noise, to that level. Since all S_I, S_H and S_M traders have the same information and strategy at period 2, we focus on their strategy at period 0 and 1.

[Insert Figure 1 Here]

Since traders submit limit orders, their orders may not be executed if limit prices cannot be met. However, the execution of orders at period $t-1$ does not influence traders' strategy at period t, because we identify traders' intended positions, instead of trading actions, in the strategy. No matter whether orders at period $t-1$ are executed or not, traders' optimal intended positions at period t will not be influenced.

2.1 Equilibrium without Communications

If traders cannot communicate with each others, they can only use their private signals and the price path, which is public information, to make their trading decisions. Their optimal strategy are shown in Lemma 1-3. Lemma 1 describes informed trader S_I's optimal strategy; Lemma 2, hybrid traders S_H; and Lemma 3, momentum traders S_M.

Lemma 1: Informed traders S_I trade only on their signals and enter the market at the very beginning if their signals are not neutral. Their optimal strategy is

$a^I(\{+\}) = \left\{a_0^I = 1, a_1^I = 1\right\};$

$a^I(\{-\}) = \{a_0^I = -1, a_1^I = -1\}$;
$a^I(\{0\}, P_1 > v_0) = \{a_0^I = 0, a_1^I = -1\}$;
$a^I(\{0\}, P_1 < v_0) = \{a_0^I = 0, a_1^I = 1\}$.

Proof: See Appendix A.

Since informed traders S_I receive perfect information about $\widetilde{v}$, they act at the very beginning if their signals are not neutral. Their optimal strategy is to benefit from their private signals immediately, i.e. to long at period 0 as soon as possible if receiving a positive signal $\{+\}$ and short as soon as possible if receiving a negative signal $\{-\}$.

[Insert Figure 2 Here]

But after they receive a neutral signal $\{0\}$, they will wait to observe price P_1 at period 1. At period 1, they may trade against those uninformed traders to make profits. At period 1, they long at price not higher than v_0 at period 1 if P_1 is lower than v_0 and short at price not lower than v_0 at period 1 if P_1 is higher than v_0. Since informed traders have perfect information about asset value and always submit limit order with limit price v_0 if receiving a neutral signal $\{0\}$, the probability that informed traders can execute their orders at period 1 is $\frac{QQ_I}{QQ_I+QQ_H}$ if they submit their orders together with hybrid traders.

Lemma 2: Hybrid traders S_H trade not only on their signals but also on the price path. They enter the market at the very beginning and decide to stay or exit after compare the price P_1 at period 1 with the threshold P_1^*. Assume $\lambda(QQ_I + QQ_H) < \widehat{v} \cdot \max\left\{p^*_{(P_1^*)}, 2p\right\}$, their optimal strategy is

$a^H(\{+,0\}, P_1 \geq v_0 + P_1^*) = \{a_0^H = 1, a_1^H = 1\}$;
$a^H(\{+,0\}, P_1 < v_0 + P_1^*) = \{a_0^H = 1, a_1^H = 0\}$;
$a^H(\{-,0\}, P_1 \leq v_0 - P_1^*) = \{a_0^H = -1, a_1^H = -1\}$;
$a^H(\{-,0\}, P_1 > v_0 - P_1^*) = \{a_0^H = -1, a_1^H = 0\}$;
where $p^* = \frac{\phi\left(\frac{P_1 - v_0 - \lambda(QQ_I+QQ_H)}{\sigma_\varepsilon}\right)\cdot 2p}{\phi\left(\frac{P_1 - v_0 - \lambda(QQ_I+QQ_H)}{\sigma_\varepsilon}\right)\cdot 2p + \phi\left(\frac{P_1 - v_0 - \lambda QQ_H}{\sigma_\varepsilon}\right)\cdot(1-2p)}$ and $P_1^* = \lambda\left(QQ_H + \frac{QQ_I}{2}\right) - \frac{\sigma_\varepsilon^2}{\lambda QQ_I} \cdot \ln\left(\frac{2p(1-K)}{K(1-2p)}\right)$

Proof: See Appendix A.

Hybrid traders S_H use not only their private signals but also public information, the price path, to make trading decisions.

For S_H, signal $\{+,0\}$ excludes state ω^- which occurs with the possibility p and signal $\{-,0\}$ excludes state ω^+ which also occurs with the possibility p. Thus, without trading costs, hybrid traders enter the market at the very beginning, as informed traders S_I in order to avoid the possible price impact from momentum traders.

However, since they have only imperfect information in their signal, to confirm their estimate, they need the price P_1 to pass the threshold P_1^*. If the price P_1 does not pass the threshold, they have to exit their positions; and if P_1 passes, they hold their position tightly because the price path confirms their signal.

[Insert Figure 3 Here]

Lemma 3: Momentum traders S_M trade only on public information, the price path. They enter market later than informed traders S_I and hybrid traders S_H. And they optimally follow the momentum/trend of the price path. Assume $\lambda\left(QQ_I + QQ_H + QQ_M\right) < \widehat{v}$, their optimal strategy is:

$a^M\left(P_1 > v_0 + P_1^{**}\right) = \left\{a_0^M = 0, a_1^M = 1\right\}$;
$a^M\left(P_1 < v_0 - P_1^{**}\right) = \left\{a_0^M = 0, a_1^M = -1\right\}$;
$a^M\left(v_0 - P_1^{**} \leq P_1 \leq v_0 + P_1^{**}\right) = \left\{a_0^M = 0, a_1^M = 0\right\}$;
where $P_1^{**} = -\frac{\sigma_\varepsilon^2}{\lambda(QQ_I+QQ_H)} \cdot \ln\left(\sqrt{\frac{1-K_1}{1+K_1} + \left(\frac{(1-2p)K_1}{2p(1+K_1)}\right)^2 \cdot e^{\frac{\lambda^2(QQ_I+QQ_H)^2}{4\sigma_\varepsilon^4}}} - \frac{(1-2p)K_1}{2p(1+K_1)} \cdot e^{-\frac{\lambda^2(QQ_I+QQ_H)^2}{2\sigma_\varepsilon^2}}\right)$

Proof: See Appendix A.

Momentum traders S_M rely on the price path to make trading decisions. S_M never trade at the very beginning because they only have uninformative signals. They infer informed traders S_I's and hybrid traders S_H's actions from the price path and make their trading decisions based on this. If the price passes the threshold P_1^{**}, then they optimally follow informed and hybrid traders to enter the market; and if not, they do not trade.

If $\frac{2p(1-K)}{K(1-2p)} \cdot \frac{1+K_1}{1-K_1} > e^{\frac{\lambda^2(QQ_I+QQ_H)^2}{\sigma_\varepsilon^2}}$, we have $P_1^{**} > P_1^*$. Momentum traders' signal is less informative than hybrid traders' and thus, in order to confirm their estimate, momentum traders need the price walk further to pass a higher threshold, given that noise σ_ε in the price cannot be ignored in S_I and S_H's price impact $\lambda\left(QQ_I + QQ_H\right)$.

[Insert Figure 4 Here]

3. Equilibrium with Communications

Now, suppose some individual traders form a group with free entries and unique identities, where traders can exchange trading, fundamental, non-fundamental and other information with each other without any cost. And such a group is unknown to or ignored by other traders outside the group.

The number of informed, hybrid and momentum traders S_I, S_H and S_M in the group are Q_I, Q_H and Q_M, where $Q_I \ll QQ_I$, $Q_H \ll QQ_H$ and $Q_M \ll QQ_M$. Also assume the number of informed traders Q_I is small enough that momentum traders' inference from number of posts cannot change their expectation about the states and thus do not change the equilibrium.

Within the group, the action space is two-dimensional, including trader i's trades and posts. At period s, trader i's action is denoted as $a_s^i \in \Lambda_2 = A \times B = \{\{-1, 0, 1\} \times \{l, s, n\}\}$, where $-1, 0, 1$ are defined as previous part, and l means posting long positions, s means posting short positions, and n means not to post any position at all. Trader i's strategy in periods $s = 0, 1$ can be denoted as $a^i = \{a_0^i, b_0^i, a_1^i, b_1^i\}$.

Suppose each type of traders can only distinguish the traders who are the same skillful as them or less skillful than them. According to this assumptio, an informed trader can only distinguish if another trader is an informed trader or not; a momentum trader can only distinguish if another trader is momentum trader or not; and a hybrid trader can distinguish if another trader is an informed trader, or a hybrid trader, or a momentum trader.

Proposition 1: With the same assumption in Lemma 1, informed traders S_I's optimal strategy within the communication group is

$a^I(\{+\}) = \left\{a_0^I = 1, b_0^I = l; a_1^I = 1, b_1^I = n\right\}$;

$a^I(\{-\}) = \left\{a_0^I = -1, b_0^I = s; a_1^I = -1, b_1^I = n\right\}$;

$a^I\left(\{0\}, b_0^H = l, P_1 > v_0 - P_1^{**}\right) = \left\{a_0^I = 0, b_0^I = n; a_1^I = -1, b_1^I = s\right\}$;

$a^I\left(\{0\}, b_0^H = s, P_1 < v_0 + P_1^{**}\right) = \left\{a_0^I = 0, b_0^I = n; a_1^I = 1, b_1^I = l\right\}$;

$a^I\left(\{0\}, P_1 \leq v_0 - P_1^{**}\right) = \left\{a_0^I = 0, b_0^I = n; a_1^I = 1, b_1^I = l\right\}$;

$a^I\left(\{0\}, P_1 \geq v_0 + P_1^{**}\right) = \left\{a_0^I = 0, b_0^I = n; a_1^I = -1, b_1^I = s\right\}$;

When receiving signal $\{0\}$, excepts the above cases, S_I do not trade or post in other situations, $\left\{a_0^I = 0, b_0^I = n; a_1^I = 0, b_1^I = n\right\}$.

Proof: See Appendix B.

In state ω^{+}/ω^{-}, after building their positions, S_I should post truthfully to attract followers, which helps the asset realize the true value earlier and informed traders benefit from that because of the time discount factor.

[Insert Figure 5 Here]

In the state ω^{0}, inferring from S_H's posts, insider S_I can exclude the noise in the price and make profits from trading against all momentum traders and outside hybrid traders. Meanwhile, in neutral state, insider S_I should not post at period 0. On one side, as hybrid traders post their position, informed traders can be free riders in trading against momentum traders. On the other side, since informed traders have no information about which kind of signal hybrid traders receive, they may post differently with hybrid traders and other informed traders, which reduces their benefits.

[Insert Figure 6 Here]

It is easy to show inside S_I better off within this group.

In the $\{\{0\}, P_1 < v_0 - P_1^{**}\}$ case, since outside momentum traders short, inside informed traders long to make profits as outside informed traders, no matter what inside hybrid traders post.

Similar analysis applies to the $\{\{0\}, P_1 > v_0 + P_1^{**}\}$ case.

Proposition 2: With the same assumption in Lemma 2, hybrid traders S_H's optimal strategy within a communication group is

$a^H(\{+,0\}, b_0^I = l) = \left\{a_0^H = 1, b_0^H = l; a_2^H = 1, b_2^H = n\right\};$

$a^H(\{+,0\}, b_0^I = n) = \left\{a_0^H = 1, b_0^H = l; a_2^H = -1, b_2^H = s\right\};$

$a^H(\{-,0\}, b_0^I = s) = \left\{a_0^H = -1, b_0^H = s; a_2^H = -1, b_2^H = n\right\};$

$a^H(\{-,0\}, b_0^I = n) = \left\{a_0^H = -1, b_0^H = s; a_1^H = 1, b_1^H = l\right\};$

In cases $\left\{\{+,0\}, b_0^I = s\right\}$ and $\left\{\{-,0\}, b_0^I = l\right\}$, inside hybrid traders trade as outside hybrid traders.

Proof: See Appendix B.

Obviously, inside S_H better off within the group because of more information. After observing S_I's posts at period 0, inside S_H attain perfect information about the state. And thus, inside S_H benefit from S_I's informative posts.

At period 0, as the outside hybrid trader, inside hybrid trader choose to enter the market at the very beginning.

At period 1, if informed traders' posts confirm hybrid traders' signal, they know the state for sure. In this situation, they can hold their position tightly and avoid being influenced by the noise in the price and exiting their position wrongly.

[Insert Figure 7 Here]

At period 1, if informed traders do not post, they know it must be the neutral state ω^0, and then, they can exit and trade against outside hybrid traders and the momentum traders. This is the reason they always post their position at period 0 to attract inside momentum traders to follow.

[Insert Figure 8 Here]

Briefly speaking, hybrid traders benefit most in the chatroom.

Proposition 3: With the same assumptions in Lemma 3, in the communication group, momentum traders S_M trade on both others' posts and the price path.

$a^M\left(b_0^{-M} = l, P_1 > v_0 + P_1^{***}\right) = \left\{a_0^M = 0, b_0^M = n; a_1^M = 1, b_1^M = l\right\}$;
$a^M\left(b_0^{-M} = s, P_1 < v_0 - P_1^{***}\right) = \left\{a_0^M = 0, b_0^M = n; a_1^M = -1, b_1^M = s\right\}$;
$a^M\left(b_0^{-M} = n, P_1 > v_0 + P_1^{**}\right) = \left\{a_0^M = 0, b_0^M = n; a_1^M = 1, b_1^M = l\right\}$;
$a^M\left(b_0^{-M} = n, P_1 < v_0 - P_1^{**}\right) = \left\{a_0^M = 0, b_0^M = n; a_1^M = -1, b_1^M = s\right\}$;
where $P_1^{***} = \lambda\left(QQ_H + \frac{QQ_I}{2}\right) - \frac{\sigma_\varepsilon^2}{\lambda QQ_I} \cdot \ln\left(\frac{2p(1-K_2)}{K_2(1-2p)}\right)$.
Inside S_M do not trade or post in other situations, $\left\{a_0^M = 0, b_0^M = n; a_1^M = 0, b_1^M = n\right\}$.
When posts are contradictory with each other, inside S_M trade as outside S_M.

Proof: See Appendix B.

Here, we assume, compared with Q_H, Q_I so small that the difference in the number of posts in states ω^- or ω^+ or ω^0 change momentum traders' expectation little and thus do not change our equilibrium.

Inside S_M do not post and trade at period 0, as outside S_M, because they do not have any information in this period.

At period 1, S_M within the group face similar situations as outside S_H: with l posts, S_M can exclude state ω^-; and with s posts, S_M can exclude state ω^+. After excluding ω^- or ω^+, in order to confirm their estimate, S_M need the price path to pass the threshold. If the price passes the threshold, they follow more informed traders' posts and enter the market; else, they hold 0 position.

[Insert Figure 9 Here]

When $\frac{2p(1-K_2)}{K_2(1-2p)} \cdot \frac{1+K_1}{1-K_1} > e^{\frac{\lambda^2(QQ_I+QQ_H)^2}{\sigma_\varepsilon^2}}$, we have $P_1^{**} > P_1^{***}$. Given that noise σ_ε in the price cannot be ignored in S_I and S_H's price impact $\lambda(QQ_I + QQ_H)$, since momentum traders inside the chatroom have more information than those outside the chatroom, inside momentum traders do not need the price path walk so far to confirm their estimate and they can enter the market with a lower threshold.

We can show that S_M better off within the group. In the states ω^+ and ω^-, S_M benefit from informative posts. And S_M's loss in the state ω^0 is less than their benefits in the state ω^+ and ω^-. In short, with more information, S_M cannot worse off.

3.1 Empirical Implications

This part summarizes the observable implications in the equilibrium of the model. We have three hypothesis indicated from the equilibrium:

Hypothesis 1. *Who posts more: neither the most skillful nor the least skillful traders trade most frequently*

Signals' informativeness shows traders' skill levels. Informed traders are the most skillful traders who get perfect information from their own analysis while momentum traders are the least skillful traders who cannot get any information from their own analysis.

In the equilibrium, S_H post much more frequently than S_I and S_M. Thus, when observing the data, we should see the anti-U-shape relation between traders' skills and their trading frequencies.

Hypothesis 2. *Who follows others: The more skillful a trader is, the less frequently she follows others.*

We use a "following" trade to denote a trade which have a previous trade traded on the same direction and posted by another trader within 5 minutes. Based on this definition, in the equilibrium, S_I seldom follow while S_M frequently follow others in stock picking. Thus, when observing the data, we should see that a trader's skill is negatively related with her following frequency.

Hypothesis 3. *Who is followed: The more skillful a trader is, the more frequently she is followed by others.*

We define the trade followed by a "following" trade as a "being followed" trade. In the equilibrium, S_I are followed by S_M with higher probability than S_H. Thus, when observing the data, we should see that a trader's skill is positively related with the number of her "being followed" trades.

4. Data and Empirical Tests

4.1 Data and Environment

The second author collected the posts from the Active Trader Financial Chatroom at sporadic intervals over a four year period from 2000 to 2003. Our sample period is the most active trading month October 2000. The logs contain several interruptions when the chat client froze or when the author neglected to capture the feed. In October 2000, we have 14 trading days of information. Posts are time stamped to the minute. Trader identities are in <.>

4.1.1 Posts

The posts contain information about fundamental and technical analysis, trades, and some irrelevant information. Here is a sample chat log from 11:48 to 11:53 Eastern time on October 30, 2000.

<UofMichigan> CSCO chart support 37, can't believe we will see that
<Tommy> CSCO wants low 40's
<Fleance> CSCO selling 46
double_odds buys COVD 5 3/16
<UofMichigan> CSCO PE not looking that bad
<getnby> sells CSCO
<aim> INTC going down with CSCO
<Sodo> CSCO 46
Matrix in CSCO
<Fleance> CSCO 800,000 shares traded last min
WallStArb buys CSCO 46 1/16
buyinlow in csco
<tradem> adding csco
<DMS> buys ITRU on NEWS
double_odds sells INDG +1/2
<[MrB]> added CSCO here
<Amokk> CSCO bounce
<ghe> buys INTC
WallStArb places 46 1/8 stop on CSCO
Matrix sells some CSCO
Targetman Buys NAS-FUTURES @ 3102
Matrix buys YHOO 52
<Commonman> $35.70/share BOUT? at what PRM price?
Targetman Buys SP-FUTURES @ 1393.50
<scalper> smart move Wally
<HITTHEBID> naz looks overdone
<phishy> bvsn stoch upcross + spoos candle bottom
Targetman Buys CSCO @ 46 3/8
<Bill1> adds xxia 18 3/4

We summarize the type of posts, number of posters and frequency in Table 2.

[Insert Table 2 Here]

Although day traders trade mostly on technical analysis, those traders did post and use fundamental information in making trading decisions. They analyzed typical fundamental indicators, stock valuation, company financial status, CEO performances and product innovations. A typical fundamental post in the example log is "[11:50] <UofMichigan> CSCO PE not looking that bad," which refers to the price earnings ratio.

Most posts about stock trading are non-fundamental posts, including technical analysis and price statements mentioning the new updates on the price path. A typical technical analysis is "[11:48] <UofMichigan> CSCO chart support 37" or "[11:53] <phishy> bvsn stock upcross + spoos candle bottom"; A typical statement about price direction is "[11:50] <aim> INTC going down with CSCO", which is simply repeating the price path, which is public information.

Traders also post their trades, which gives us the information about their real skills. A typical trade post is "[11:53] Targetman Buys CSCO @ 46 3/8", in which the trader <Targetman> bought CSCO at the price he showed. We do not rely on the trader's posted price and profit information, but instead verify this from transactions records.

There are posts irrelevant with stock trading, such as "[11:53] <scalper> smart move Wally" in the sample chat log. However, since there are chatroom administrators who keep the room focus on stock trading within trading hours, most totally irrelevant posts appear after trading hours.

4.1.2 Trades

We also summarize the trading activity for October 2000 in Table 2.

Traders use a wide variety of slang for their trades. We used various forms of the keywords, including their abbreviations and misspelled variants, to indicate buying activity: Accumulate; Add; Back; Buy; Cover; Enter; Get; Grab; In; Into; Load; Long; Nibble; Nip; Pick; Poke; Reload; Take; and Try. Keywords for selling were: Dump; Out; Scalp; Sell; Short; Stop; and Purge.

We cannot match open and closing trades for about 70% of the posts. We assume that all open positions whether long or short are closed at the end of the day. We do not consider after hours trades.

4.1.3 Profits

To compute dollar profit and losses for each trader, we make transaction cost assumptions for position size assumptions. For position size A, we assume a $20 commission. This is a $0.02 per share commission on the 1,000 share round trip. Numerous brokers offer commissions in this range. For position size B, we assume a $0.005 per share commission and a 50 basis point slippage. These reflect the lower commissions typically paid on larger lot sizes, and some market impact on the larger trades. We find that none of the position or transaction costs assumptions has a qualitative impact on our profit estimates.

We examine profits for all trades. The first profit measure is the aggregate difference between selling and buying prices so the reader can gauge the effect of the transactions costs. The second measure A uses the low cost estimate with flat commissions. The second measure B has higher transactions costs, but sometimes benefits from the larger lot sizes.

In our sample period, more than 50% of traders are profitable under A while 47.48% of the traders are profitable under B. These are much higher ratios of profitable traders found in other studies of retail investors or day traders. This is why we feel comfortable regarding some semi-professional and professional traders as informed traders. The experts in our chat room are "Activetraders" for a good reason; trading, for them, is a profitable activity.

Our traders make money trading both long and short. When we break apart profits short versus long, we find that 74.7% of profits are made trading long and 25.3% short. Trades are equally likely to be profitable long versus short, 53.97% long compared to 56.07% short. The marginal profit per trade is substantially higher on the short side than the long, $210.84 per trade short versus $110.87 long in the pooled sample. Short traders are also more skillful overall. Over the four years, 51.55% of traders who never short are profitable under assumption A, compared with 62.21% for traders who trade both short and long.

For the remainder of this section, we will utilize the more conservative profit assumptions A.

4.2 Empirical Results

Hypothesis 1. *Who posts more: neither the most skillful nor the least skillful traders trade most frequently*

Our first test of the model is about posting frequency by trader j for the four types of posts: (1) fundamental posts, FP_j;(2) non-fundamental posts, NFP_j;(3) trade posts, TRP_j; (4) irrelevant posts, IRR_j. Trader j's total posts are

$$NP_j = FP_j + NFP_j + TRP_j + IRR_j. \tag{1}$$

H1 tests the posting frequency of trades, TRP_j/NP_j.

We calculate our standard skill measure, the profit per trade of trader j

$$\pi_j = \frac{\sum_{t=1}^{Tr_j} \pi_{j,t}}{\sum_{t=1}^{Tr_j} Tr_{j,t}} \tag{2}$$

And we separate all traders into two groups π_j^+ and π_j^-, where π_j^+ refer to profits of traders with positive profits and π_j^- refer to profits of traders with negative profits, and then regress π_j^+ and π_j^- respectively on the number of each type traders' trading post,

$$TRP_j = \alpha_{1A} + \beta_{1A}\pi_j^-, \tag{3}$$

and

$$TRP_j = \alpha_{1B} + \beta_{1B}\pi_j^+, \tag{4}$$

We find statistically significant $\beta_{1A} > 0$ and $\beta_{1B} < 0$ in Table 3. $\beta_{1A} > 0$ shows, within traders with negative profits, the more skillful a trader is, the less frequently she posts trades. And so, the middle skill level traders post trades more frequently than the low skill level traders. $\beta_{1B} < 0$ shows, within traders with positive profits, the more skillful a trader is, the more frequently she posts trades. And so, the middle skill level traders also post trades more frequently than the high skill level traders. Therefore, the empirical results show the middle skill level traders post trades most often within the group.

Hypothesis 2. *Who follows others: The more skillful a trader is, the less frequently she follows others.*

We first test hypothesis H2a: The more skillful a trader is, the less likely she will follow others. We partition trade profits into following and non-following, $\pi_j = \pi_j^{(f)} + \pi_j^{(nf)}$, using profits obtained while not following as a skill measure. We regress the following rate, $F_j = TR_j^{(f)}/(TR_j^{(f)} + TR_j^{(nf)})$, on profits per non-following trade $\pi_j^{(f)}$on

$$F_j = \alpha_{2a} + \beta_{2a}\pi_j \ . \tag{5}$$

We find that β_{2a} is significantly less than zero, consistent with the hypothesis.

We next test hypothesis H2b: Do unskilled traders benefit more from following. We consider trades where an unskillful trader $\pi_j < 0$ follows a skillful trader, $\pi_j > 0$. We partition trade profits into following and non-following, $\pi_j = \pi_j^{(f)} + \pi_j^{(nf)}$ and regress total profits on the difference,

$$\pi_j^{(f)} - \pi_j^{(nf)} = \alpha_{2b} + \beta_{2b}\pi_j \ . \tag{6}$$

We find that $\beta_{2b} < 0$.

$\beta_{2a} < 0$ and $\beta_{2b} < 0$ shows traders' skills are negatively related with their following frequency and their profits from following. The more skillful a trader is, the less frequently she follows others.

Hypothesis 3. *Who is followed: The more skillful a trader is, the more frequently she is followed by others.*

Hypothesis 3 asks whether skillful traders have more followers? Define trader j's total trades and her trades followed by traders other than j as Tr_j and $Tr_{-j}^{(f)}$, and define the being followed rate,

$$F_{-j} = Tr_{-j}^{(f)}/Tr_j \tag{7}$$

We then regress the skill level on the "being followed" rate,

$$F_{-j} = \alpha_3 + \beta_3 \pi_j \ . \tag{8}$$

and find that $\beta_3 > 0$, indicating strong support of the hypothesis.

$\beta_3 > 0$ shows traders' skills are positively related with their being-followed rate. The more skillful a trader is, the more frequently she is followed by others.

5. Conclusions and Extensions

This paper studies individual day traders and their communications. An interaction game is built up to explain individual traders' strategic behaviors in an internet stock trading chat room. And we model how communications influence traders' trading decisions and explain how the chat room is beneficial to all participants, even the most skillful traders. Informed traders benefit from trading against momentum traders. Hybrid traders benefit from both informed traders' informative posts and trading against momentum traders. Momentum traders benefit from informative posts in the group.

We motivate three empirical results: (1) Neither the most informed nor the most uninformed traders communicates most often; (2) Both hybrid and momentum traders learn from public information about prices; and (3) They optimally follow informed traders. And we do find out that traders have some knowledge of who the skillful traders are and follow more often the most skillful traders, instead of the most active ones.

It is interesting to speculate whether Wall Street is just a large version of the chatroom. For example, large financial institutions are doing two things which skillful traders did in this chat room: (1) building positions before releasing information (see e.g. Mizrach (2005); and (2) taking advantage of reputation as was disclosed in Elliot Spitzer's investigations in 2002.

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Appendix A

Proof of Lemmas

For hybrid traders S_H:

$$\begin{aligned}
\Pr[\omega^+|\{+,0\}] &= \frac{\Pr[\{+,0\}|\omega^+]\cdot\Pr[\omega^+]}{\Pr[\{+,0\}|\omega^+]\cdot\Pr[\omega^+]+\Pr[\{+,0\}|\omega^0]\cdot\Pr[\omega^0]+\Pr[\{+,0\}|\omega^-]\cdot\Pr[\omega^-]}, \\
&= \frac{1\cdot p}{1\cdot p+\frac{1}{2}\cdot(1-2p)+0} \\
&= 2p
\end{aligned}$$

and $\Pr[\omega^0|\{+,0\}]=1-2p$, $\Pr[\omega^-|\{+,0\}]=0$.

Similarly, $\Pr[\omega^-|\{-,0\}]=2p$, $\Pr[\omega^0|\{-,0\}]=1-2p$, $\Pr[\omega^+|\{-,0\}]=0$.

We assume $2p\widehat{v}>\lambda(QQ_I+QQ_H)$ to simplify the problem but loosing this assumption does not influence the communication effects.

After observing price at period 1,

$$\begin{aligned}
&\Pr[\omega^+|\{+,0\},P_1] \\
=& \frac{\Pr[P_1|\{+,0\},\omega^+]\cdot\Pr[\omega^+|\{+,0\}]}{\Pr[P_1|\{+,0\},\omega^+]\cdot\Pr[\omega^+|\{+,0\}]+\Pr[P_1|\{+,0\},\omega^0]\cdot\Pr[\omega^0|\{+,0\}]+\Pr[P_1|\{+,0\},\omega^-]\cdot\Pr[\omega^-|\{+,0\}]} \\
=& \frac{\Pr[P_1|\{+,0\},\omega^+]\cdot 2p}{\Pr[P_1|\{+,0\},\omega^+]\cdot 2p+\Pr[P_1|\{+,0\},\omega^0]\cdot(1-2p)} \\
=& \frac{\Pr[\varepsilon=P_1-v_0-\lambda(QQ_I+QQ_H)|\omega^+]\cdot 2p}{\Pr[\varepsilon=P_1-v_0-\lambda(QQ_I+QQ_H)|\omega^+]\cdot 2p+\Pr[\varepsilon=P_1-v_0-\lambda QQ_H|\omega^0]\cdot(1-2p)} \\
=& \frac{\phi\left(\frac{P_1-v_0-\lambda(QQ_I+QQ_H)}{\sigma_\varepsilon}\right)\cdot 2p}{\phi\left(\frac{P_1-v_0-\lambda(QQ_I+QQ_H)}{\sigma_\varepsilon}\right)\cdot 2p+\phi\left(\frac{P_1-v_0-\lambda QQ_H}{\sigma_\varepsilon}\right)\cdot(1-2p)} \\
\triangleq& p^*
\end{aligned}$$

To simplify the problem, we regard $v_0=0$ which does not influence the conclusions.

At period 0, S_H's expected returns:

$$E[\pi_H\left(a_0^H=0,a_1^H=0\right)|\{+,0\}]=0$$

$$
\begin{aligned}
& E[\pi_H\left(a_0^H=0, a_1^H=1\right) \mid \{+,0\}] \\
= \;& 2p\left\{0+\beta\lambda\left(\frac{QQ_H}{2}\right)+\beta^2\left[\widehat{v}-\lambda\left(QQ_I+QQ_H\right)\right]\right\} \\
& +(1-2p)\left\{0+\beta\lambda\left(\frac{QQ_H}{2}\right)+\beta^2\cdot\left(-\lambda QQ_H\right)\right\} \\
= \;& \lambda\frac{QQ_H}{2}\left(\beta-2\beta^2\right)+2p\left\{\beta^2\left[\widehat{v}-\lambda QQ_I\right]\right\}
\end{aligned}
$$

$$
\begin{aligned}
& E[\pi_H\left(a_0^H=1, a_1^H=0\right) \mid \{+,0\}] \\
= \;& 2p\left\{\lambda\left(\frac{QQ_H}{2}+\frac{QQ_I}{2}\right)+\beta\lambda\left(-\frac{QQ_H}{2}\right)+\beta^2\cdot 0\right\} \\
& +(1-2p)\left\{\lambda\left(\frac{QQ_H}{2}\right)+\beta\left[\frac{QQ_H}{QQ_I+QQ_H}\cdot\left(-\lambda\frac{QQ_H}{2}\right)\right]+\beta^2\cdot\left[\frac{QQ_I}{QQ_I+QQ_H}\left(-\lambda QQ_H\right)\right]\right\} \\
= \;& \lambda\frac{QQ_H}{2}\left(1-\beta\frac{QQ_H}{QQ_I+QQ_H}-2\beta^2\frac{QQ_I}{QQ_I+QQ_H}\right)+2p\left\{\lambda\frac{QQ_I}{2}-\lambda\beta\cdot\frac{QQ_I}{2}\left(\frac{QQ_H}{QQ_I+QQ_H}\right)(1-2\beta)\right\}
\end{aligned}
$$

$$
\begin{aligned}
& E[\pi_H\left(a_0^H=1, a_1^H=1\right) \mid \{+,0\}] \\
= \;& 2p\left\{\lambda\left(\frac{QQ_I}{2}+\frac{QQ_H}{2}\right)+\beta^2\left[\widehat{v}-\lambda\left(QQ_I+QQ_H\right)\right]\right\} \\
& +(1-2p)\left\{\lambda\left(\frac{QQ_H}{2}\right)+\beta^2\left(-\lambda QQ_H\right)\right\} \\
= \;& \lambda\frac{QQ_H}{2}\left(1-2\beta^2\right)+2p\left\{\lambda\frac{QQ_I}{2}+\beta^2\left[\widehat{v}-\lambda QQ_I\right]\right\}
\end{aligned}
$$

Here, $a_0^H=1, a_1^H=1$ means hybrid traders send an order to buy at price not higher than their expectation $2p\widehat{v}$ at period 0 and hold the long position if the orders are executed; $a_0^H=1, a_1^H=0$ means hybrid traders send an order to buy at price not higher than $2p\widehat{v}$ at period 0 and sell the long position at price not lower than 0 at period 1 if the orders at period 0 are executed; $a_0^H=0, a_1^H=1$ means hybrid traders hold position 0 at period 0 and send an order to buy at price not higher than $p^*\widehat{v}$ at period 1; $a_0^H=0, a_1^H=0$ means hybrid traders do not buy or sell at all. Since signal $\{+,0\}$ exclude state ω^-, hybrid traders do not short in this situation.

Since $\beta<1$, we can easily attain that $E[\pi_H\left(a_0^H=1, a_1^H=1\right) \mid \{+,0\}] > E[\pi_H\left(a_0^H=0, a_1^H=1\right) \mid \{+,0\}]$ always holds under any situation.

Since $2p\widehat{v}>\lambda\left(QQ_I+QQ_H\right)$, $\widehat{v}>\frac{\lambda}{2p}\left(QQ_I+QQ_H\right)>\frac{\lambda}{2p}\left(1-\frac{\beta^2}{2}\right)\left(2p\cdot QQ_I+QQ_H\right)$ and we can get

$$E[\pi_H\left(a_0^H=1, a_1^H=1\right) \mid \{+,0\}] > E[\pi_H\left(a_0^H=0, a_1^H=0\right) \mid \{+,0\}] = 0$$

Besides, it is easy to see that shorting at period 0 / $a_0^H=-1$ yields negative expected returns when S_H receive positive signal $\{+,0\}$.

Therefore, at period 0, longing / $a_0^H = 1$ is always the optimal choice for S_H.

At period 1, S_H's expected returns:

$$
\begin{aligned}
& E[\pi_H\left(a_0^H = 1, a_1^H = 0\right) \mid \{+,0\}, P_1] \\
= \; & p^* \left\{ \lambda \left(\frac{QQ_H}{2} + \frac{QQ_I}{2} \right) + \beta\lambda \left(-\frac{QQ_H}{2} \right) + \beta^2 \cdot 0 \right\} \\
& + (1-p^*) \left\{ \lambda \left(\frac{QQ_H}{2} \right) + \beta \left[\frac{QQ_H}{QQ_I + QQ_H} \cdot \left(-\lambda \frac{QQ_H}{2} \right) \right] + \beta^2 \cdot \left[\frac{QQ_I}{QQ_I + QQ_H} \left(-\lambda QQ_H \right) \right] \right\} \\
= \; & \lambda \frac{QQ_H}{2} \left(1 - \beta \frac{QQ_H}{QQ_I + QQ_H} - 2\beta^2 \frac{QQ_I}{QQ_I + QQ_H} \right) \\
& + p^* \left\{ \lambda \frac{QQ_I}{2} - \lambda\beta \cdot \frac{QQ_I}{2} \left(\frac{QQ_H}{QQ_I + QQ_H} \right) (1 - 2\beta) \right\}
\end{aligned}
$$

$$
\begin{aligned}
& E[\pi_H\left(a_0^H = 1, a_1^H = 1\right) \mid \{+,0\}, P_1] \\
= \; & p^* \left\{ \lambda \left(\frac{QQ_I}{2} + \frac{QQ_H}{2} \right) + \beta^2 \left[\widehat{v} - \lambda \left(QQ_I + QQ_H \right) \right] \right\} \\
& + (1-p^*) \left\{ \lambda \left(\frac{QQ_H}{2} \right) + \beta^2 \left(-\lambda QQ_H \right) \right\} \\
= \; & \lambda \frac{QQ_H}{2} \left(1 - 2\beta^2 \right) + p^* \left\{ \lambda \frac{QQ_I}{2} + \beta^2 \left[\widehat{v} - \lambda QQ_I \right] \right\}
\end{aligned}
$$

Let us compare the two strategies:

$$
\begin{aligned}
& E[\pi_H\left(a_0^H = 1, a_1^H = 1\right) \mid \{+,0\}, P_1] - E[\pi_H\left(a_0^H = 1, a_1^H = 0\right) \mid \{+,0\}, P_1] \\
= \; & \lambda\beta \frac{QQ_H}{2} \frac{QQ_H}{QQ_I + QQ_H} (1 - 2\beta) + p^* \left\{ \beta^2 \widehat{v} - \lambda\beta \frac{QQ_I}{2} \left[2\beta + (2\beta - 1) \frac{QQ_H}{QQ_I + QQ_H} \right] \right\}
\end{aligned}
$$

$$
\begin{aligned}
& E[\pi_H\left(a_0^H = 1, a_1^H = 1\right) \mid \{+,0\}, P_1] > E[\pi_H\left(a_0^H = 1, a_1^H = 0\right) \mid \{+,0\}, P_1] \\
\Longrightarrow \; & p^* > \frac{\lambda \frac{QQ_H}{2} \frac{QQ_H}{QQ_I + QQ_H} (2\beta - 1)}{\beta^2 \widehat{v} - \lambda\beta \frac{QQ_I}{2} \left[2\beta + (2\beta - 1) \frac{QQ_H}{QQ_I + QQ_H} \right]} \\
\Longrightarrow \; & \frac{\phi\left(\frac{P_1 - v_0 - \lambda(QQ_I + QQ_H)}{\sigma_\varepsilon} \right) \cdot 2p}{\phi\left(\frac{P_1 - v_0 - \lambda(QQ_I + QQ_H)}{\sigma_\varepsilon} \right) \cdot 2p + \phi\left(\frac{P_1 - v_0 - \lambda QQ_H}{\sigma_\varepsilon} \right) \cdot (1 - 2p)} > \frac{\lambda \frac{QQ_H}{2} \frac{QQ_H}{QQ_I + QQ_H} (2\beta - 1)}{\beta^2 \widehat{v} - \lambda\beta \frac{QQ_I}{2} \left[2\beta + (2\beta - 1) \frac{QQ_H}{QQ_I + QQ_H} \right]}
\end{aligned}
$$

Denote $\frac{\lambda \frac{QQ_H}{2} \frac{QQ_H}{QQ_I + QQ_H} (2\beta - 1)}{\beta^2 \widehat{v} - \lambda\beta \frac{QQ_I}{2} \left[2\beta + (2\beta - 1) \frac{QQ_H}{QQ_I + QQ_H} \right]} = K$. With $2p\widehat{v} > \lambda\left(QQ_I + QQ_H\right)$ and assume $\beta > \frac{1}{2}$, we can easily have $0 < K < 1$.

Thus,

$$
\begin{aligned}
E[\pi_H\left(a_0^H=1,a_1^H=1\right)|\{+,0\},P_1] &> E[\pi_H\left(a_0^H=1,a_1^H=0\right)|\{+,0\},P_1] \\
&\Longrightarrow P_1 > v_0 + P_1^* \\
\text{, where } P_1^* &= \lambda\left(QQ_H+\frac{QQ_I}{2}\right)-\frac{\sigma_\varepsilon^2}{\lambda QQ_I}\cdot\ln\left(\frac{2p\,(1-K)}{K\,(1-2p)}\right)
\end{aligned}
$$

Hybrid trader's optimal strategy is to buy the risky asset at the price not higher than $2p\widehat{v}$ at period 0, and then to hold the long position if price at period 1 passes the threshold P_1^*; otherwise, exit the position at period 1.

We also need to assume $p^*_{(v_0+P_1^*)}\cdot\widehat{v}>\lambda\,(QQ_I+QQ_H)$.

Let's consider momentum traders S_M.

Since momentum traders do not receive any informative signal, i.e. $\Pr[\omega^+|\{+,0,-\}]=p=\Pr[\omega^-|\{+,0,-\}]$, obviously, the optimal choice for momentum traders at period 0 is to do nothing, i.e. $a_0^M=0$ holds in all situations.

We assume $\widehat{v}>\lambda\,(QQ_I+QQ_H+QQ_M)$ to simplify the problem but loosing this assumption does not influence the communication effects.

Then, at period 1,

$\Pr[\omega^+|\{+,0,-\},P_1]$

$$
\begin{aligned}
&= \frac{\Pr[P_1|\{+,0,-\},\omega^+]\cdot\Pr[\omega^+|\{+,0,-\}]}{\left\{\begin{array}{c}\Pr[P_1|\{+,0,-\},\omega^+]\cdot\Pr[\omega^+|\{+,0,-\}]+\Pr[P_1|\{+,0,-\},\omega^0]\cdot\Pr[\omega^0|\{+,0,-\}]\\ +\Pr[P_1|\{+,0,-\},\omega^-]\cdot\Pr[\omega^-|\{+,0,-\}]\end{array}\right\}} \\
&= \frac{\Pr[P_1|\{+,0,-\},\omega^+]\cdot p}{\Pr[P_1|\{+,0,-\},\omega^+]\cdot p+\Pr[P_1|\{+,0,-\},\omega^0]\cdot(1-2p)+\Pr[P_1|\{+,0,-\},\omega^-]\cdot p} \\
&= \frac{\phi\left(\frac{P_1-v_0-\lambda(QQ_I+QQ_H)}{\sigma_\varepsilon}\right)\cdot p}{\phi\left(\frac{P_1-v_0-\lambda(QQ_I+QQ_H)}{\sigma_\varepsilon}\right)\cdot p+\phi\left(\frac{P_1-v_0}{\sigma_\varepsilon}\right)\cdot(1-2p)+\phi\left(\frac{P_1-v_0+\lambda(QQ_I+QQ_H)}{\sigma_\varepsilon}\right)\cdot p} \\
&\triangleq p^+
\end{aligned}
$$

Similarly,

$$
\begin{aligned}
&\Pr[\omega^0 | \{+,0,-\}, P_1] \\
= &\ \frac{\phi\left(\frac{P_1-v_0}{\sigma_\varepsilon}\right)\cdot(1-2p)}{\phi\left(\frac{P_1-v_0-\lambda(QQ_I+QQ_H)}{\sigma_\varepsilon}\right)\cdot p+\phi\left(\frac{P_1-v_0}{\sigma_\varepsilon}\right)\cdot(1-2p)+\phi\left(\frac{P_1-v_0+\lambda(QQ_I+QQ_H)}{\sigma_\varepsilon}\right)\cdot p} \\
\triangleq &\ p^0
\end{aligned}
$$

$$
\begin{aligned}
\Pr[\omega^- | \{+,0,-\}, P_1] &= \frac{\phi\left(\frac{P_1-v_0+\lambda(QQ_I+QQ_H)}{\sigma_\varepsilon}\right)\cdot p}{\phi\left(\frac{P_1-v_0-\lambda(QQ_I+QQ_H)}{\sigma_\varepsilon}\right)\cdot p+\phi\left(\frac{P_1-v_0}{\sigma_\varepsilon}\right)\cdot(1-2p)+\phi\left(\frac{P_1-v_0+\lambda(QQ_I+QQ_H)}{\sigma_\varepsilon}\right)\cdot p} \\
&\triangleq p^-
\end{aligned}
$$

At period 1, momentum traders' expected returns:

$$
E[\pi_M\left(a_0^M=0, a_1^M=0\right) | \{+,0,-\}, P_1] = 0
$$

$$
\begin{aligned}
&E[\pi_M\left(a_0^M=0, a_1^M=1\right) | \{+,0,-\}, P_1] \\
= &\ p^+\left\{0+\beta\lambda\left(\frac{QQ_M}{2}\right)+\beta^2\left[\widehat{v}-\lambda\left(QQ_I+QQ_H+QQ_M\right)\right]\right\} \\
&+p^0\left\{0+\beta\lambda\left(\frac{QQ_M}{2}\right)+\beta^2\cdot\left[-\lambda QQ_M\right]\right\} \\
&+p^-\left\{0+\beta\lambda\left(\frac{QQ_M}{2}\right)+\beta^2\left[-\widehat{v}+\lambda\left(QQ_I+QQ_H-QQ_M\right)\right]\right\} \\
= &\ \lambda\beta\frac{QQ_M}{2}(1-2\beta)+\left(p^+-p^-\right)\beta^2\left\{\widehat{v}-\lambda\beta\left(QQ_I+QQ_H\right)\right\}
\end{aligned}
$$

$$
\begin{aligned}
&E\left[\pi_M\left(a_0^M=0, a_1^M=-1\right) | \{+,0,-\}, P_1\right] \\
= &\ p^+\left\{0+\beta\lambda\left(\frac{QQ_M}{2}\right)+\beta^2\left[-\widehat{v}+\lambda\left(QQ_I+QQ_H-QQ_M\right)\right]\right\} \\
&+p^0\left\{0+\beta\lambda\left(\frac{QQ_M}{2}\right)+\beta^2\cdot\left[-\lambda QQ_M\right]\right\} \\
&+p^-\left\{0+\beta\lambda\left(\frac{QQ_M}{2}\right)+\beta^2\left[\widehat{v}-\lambda\left(QQ_I+QQ_H+QQ_M\right)\right]\right\} \\
= &\ \lambda\beta\frac{QQ_M}{2}(1-2\beta)+\left(p^--p^+\right)\beta^2\left\{\widehat{v}-\lambda\beta\left(QQ_I+QQ_H\right)\right\}
\end{aligned}
$$

To compare the strategies:

$$
\begin{aligned}
& E[\pi_M\left(a_0^M = 0, a_1^M = 1\right) \mid \{+, 0, -\}, P_1] \\
> \quad & E[\pi_M\left(a_0^M = 0, a_1^M = 0\right) \mid \{+, 0, -\}, P_1] = 0 \\
\Longrightarrow \quad & \left(p^+ - p^-\right) \beta^2 \left\{\widehat{v} - \lambda\beta\left(QQ_I + QQ_H\right)\right\} > \lambda\beta \frac{QQ_M}{2}\left(2\beta - 1\right)
\end{aligned}
$$

Similarly,

$$
\begin{aligned}
& E[\pi_M\left(a_0^M = 0, a_1^M = -1\right) \mid \{+, 0, -\}, P_1] \\
> \quad & E[\pi_M\left(a_0^M = 0, a_1^M = 0\right) \mid \{+, 0, -\}, P_1] = 0 \\
\Longrightarrow \quad & \left(p^- - p^+\right) \beta^2 \left\{\widehat{v} - \lambda\beta\left(QQ_I + QQ_H\right)\right\} > \lambda\beta \frac{QQ_M}{2}\left(2\beta - 1\right)
\end{aligned}
$$

Denote $\frac{\lambda QQ_M\left(1-\frac{1}{2\beta}\right)}{\widehat{v}-\lambda\beta(QQ_I+QQ_H)} = K_1$. Since $\widehat{v} > \lambda\left(QQ_I + QQ_H + QQ_M\right)$, $0 < K_1 < 1$.

Thus,

$$
\begin{aligned}
& E[\pi_M\left(a_0^M = 0, a_1^M = 1\right) \mid \{+, 0, -\}, P_1] \\
> \quad & E[\pi_M\left(a_0^M = 0, a_1^M = 0\right) \mid \{+, 0, -\}, P_1] = 0 \\
\Longrightarrow \quad & P_1 > v_0 + P_1^{**}
\end{aligned}
$$

$$
\text{where } P_1^{**} \quad = \quad -\frac{\sigma_\varepsilon^2}{\lambda\left(QQ_I + QQ_H\right)} \cdot \ln\left(\sqrt{\frac{1-K_1}{1+K_1} + \left(\frac{(1-2p)K_1}{2p\left(1+K_1\right)}\right)^2 \cdot e^{\frac{\lambda^2\left(QQ_I+QQ_H\right)^2}{4\sigma_\varepsilon^4}}} - \frac{(1-2p)K_1}{2p\left(1+K_1\right)} \cdot e^{-\frac{\lambda^2\left(QQ_I+Q\right.}{2\sigma_\varepsilon^2}}\right.
$$

Similarly,

$$
\begin{aligned}
& E[\pi_M\left(a_0^M = 0, a_1^M = -1\right) \mid \{+, 0, -\}, P_1] \\
> \quad & E[\pi_M\left(a_0^M = 0, a_1^M = 0\right) \mid \{+, 0, -\}, P_1] = 0 \\
\Longrightarrow \quad & P_1 < v_0 - P_1^{**}
\end{aligned}
$$

Here $a_0^M = 0, a_1^M = 1$ means momentum traders hold position 0 at period 0 and submit an order to buy at price not higher than $p^+\widehat{v}$ at period 1; $a_0^M = 0, a_1^M = 0$ means momentum traders do not buy or sell at all; $a_0^M = 0, a_1^M = -1$ means momentum traders hold position 0 at period 0 and submit an order to sell at price not higher than $p^-\widehat{v}$ at period 1.

If $\frac{2p(1-K)}{K(1-2p)} \cdot \frac{1+K_1}{1-K_1} > e^{\frac{\lambda^2\left(QQ_I+QQ_H\right)^2}{\sigma_\varepsilon^2}}$, we have $P_1^{**} > P_1^*$. Momentum traders' signal is less informative than hybrid traders' and thus, they need the price path walk further to confirm the trend, if noise σ_ε in the price cannot be ignored in S_I and S_H's price impact $\lambda\left(QQ_I + QQ_H\right)$.

For informed traders S_I:

Obviously, S_I's optimal strategy is $\left\{a_0^I = 1, a_1^I = 1\right\}$ if receiving $\{+\}$ and $\left\{a_0^I = -1, a_1^I = -1\right\}$ if receiving $\{-\}$.

$$\begin{aligned}
& E[\pi_I \left(a_0^I = 1, a_1^I = 1\right) | \{+\}] \\
= \;& E[\pi_I \left(a_0^I = -1, a_1^I = -1\right) | \{-\}] \\
= \;& \lambda \left(\frac{QQ_I}{2} + \frac{QQ_H}{2}\right) + \beta \cdot \Pr\left[\varepsilon > P_1^{**} - \lambda\left(QQ_I + QQ_H\right)\right] \cdot \left(\lambda QQ_M\right) \\
& + \beta^2 \left[\widehat{v} - \lambda\left(QQ_I + QQ_H + \Pr\left[\varepsilon > P_1^{**} - \lambda\left(QQ_I + QQ_H\right)\right] QQ_M\right)\right] \\
= \;& \beta^2 \widehat{v} + \lambda \left(\frac{QQ_I}{2} + \frac{QQ_H}{2}\right)\left(1 - 2\beta^2\right) + \lambda\beta QQ_M \left(1 - \beta\right)\left[1 - \Phi\left(P_1^{**} - \lambda\left(QQ_I + QQ_H\right)\right)\right] \\
> \;& 0
\end{aligned}$$

When receiving a signal $\{0\}$:

$$\begin{aligned}
& \Pr\left\{S_H = \{+, 0\} | P_1, \omega^0\right\} \\
= \;& \frac{\phi\left(\frac{P_1 - v_0 - \lambda QQ_H}{\sigma_\varepsilon}\right) \cdot \frac{1}{2} \cdot (1 - 2p)}{\phi\left(\frac{P_1 - v_0 - \lambda QQ_H}{\sigma_\varepsilon}\right) \cdot \frac{1}{2} \cdot (1 - 2p) + \phi\left(\frac{P_1 - v_0 + \lambda QQ_H}{\sigma_\varepsilon}\right) \cdot \frac{1}{2} \cdot (1 - 2p)} \\
\triangleq \;& p^{**}
\end{aligned}$$

and

$$\Pr\left\{S_H = \{-, 0\} | P_1, \omega^0\right\} = 1 - p^{**}$$

When $P_1 > v_0$, we have $p^{**} > \frac{1}{2} > 1 - p^{**}$; and when $P_1 < v_0$, we have $p^{**} < \frac{1}{2} < 1 - p^{**}$.

When $v_0 < P_1 < v_0 + P_1^*$, S_I's expected returns are:

$$\begin{aligned}
& E[\pi_I \left(a_0^I = 0, a_1^I = -1\right) | \{0\}, v_0 < P_1 < v_0 + P_1^*] \\
= \;& p^{**}\left\{\lambda\beta\left(\frac{QQ_H}{2} \cdot \frac{QQ_I}{QQ_I + QQ_H}\right)\right\}
\end{aligned}$$

and

$$\begin{aligned}
& E[\pi_I \left(a_0^I = 0, a_1^I = 1\right) | \{0\}, v_0 < P_1 < v_0 + P_1^*] \\
= \;& \left(1 - p^{**}\right)\left\{\lambda\beta\left(\frac{QQ_H}{2} \cdot \frac{QQ_I}{QQ_I + QQ_H}\right)\right\}
\end{aligned}$$

Here, $a_0^I = 0, a_1^I = 1$ means informed traders submit no order at period 0 and submit an order to buy at price not higher than v_0 at period 1; $a_0^I = 0, a_1^I = -1$ means informed traders submit no order at period 0 and submit an order to sell at price not lower than v_0 at period 1.

Since $P_1 > v_0 \Longrightarrow p^{**} > \frac{1}{2} > 1 - p^{**}$,

$$\begin{aligned} E[\pi_I\left(a_0^I = 0, a_1^I = -1\right) \mid \{0\}, v_0 &< P_1 < v_0 + P_1^*] \\ &> E[\pi_I\left(a_0^I = 0, a_1^I = 1\right) \mid \{0\}, v_0 < P_1 < v_0 + P_1^*] \end{aligned}$$

When $P_1 > v_0 + P_1^*$, S_I's expected returns are:

$$\begin{aligned} & E[\pi_I\left(a_0^I = 0, a_1^I = -1\right) \mid \{0\}, P_1 > v_0 + P_1^*] \\ = \;& p^{**}\left\{\lambda\beta\left(QQ_H - \frac{QQ_I}{2}\right)\right\} \end{aligned}$$

and

$$\begin{aligned} & E[\pi_I\left(a_0^I = 0, a_1^I = 1\right) \mid \{0\}, P_1 < v_0 + P_1^*] \\ = \;& (1 - p^{**})\left\{\lambda\beta\left(QQ_H - \frac{QQ_I}{2}\right)\right\} \end{aligned}$$

Since $P_1 > v_0 + P_1^* \Longrightarrow p^{**} > \frac{1}{2} > 1 - p^{**} \Longrightarrow p^{**} > \frac{1}{2}$,

$$\begin{aligned} E[\pi_I\left(a_0^I = 0, a_1^I = -1\right) \mid \{0\}, P_1 &> v_0 + P_1^*] \\ &> E[\pi_I\left(a_0^I = 0, a_1^I = 1\right) \mid \{0\}, P_1 > v_0 + P_1^*] \end{aligned}$$

Similarly, we can show, when $P_1 < v_0$,

$$\begin{aligned} E[\pi_I\left(a_0^I = 0, a_1^I = -1\right) \mid \{0\}, v_0 &< P_1 < v_0 + P_1^*] \\ &< E[\pi_I\left(a_0^I = 0, a_1^I = 1\right) \mid \{0\}, v_0 < P_1 < v_0 + P_1^*] \end{aligned}$$

So, if receiving signal $\{0\}$, informed traders do not trade at period 0, and then, they long at price not higher than v_0 at period 1 if P_1 is lower than v_0 and short at price not lower than v_0 at period 1 if P_1 is higher than v_0 . Since informed traders have perfect information about asset value and submit limit order with limit price v_0, the probability that informed traders can execute their orders at period 1 is $\frac{QQ_I}{QQ_I + QQ_H}$ if they submit their orders together with hybrid traders. Meanwhile, in this case, hybrid traders can exit from their positions with probability $\frac{QQ_H}{QQ_I + QQ_H}$ at period 1 if they submit their orders together with informed traders.

Appendix B

Proof of Proposition

Within this group, we suppose each type of traders can only distinguish the traders who are the same skillful as them or less skillful than them.. And we also assume the number of informed traders Q_I is small enough that momentum traders' inference from number of posts cannot change their expectation about the states and thus do not change our equilibrium.

It is easy to show informed traders better off by posting truthfully in state ω^- and ω^+:

$$\begin{aligned}
& E[\pi_I\left(a_0^I = 1, b_0^I = n, a_1^I = 1, b_1^I = n\right) \mid \{+\}] \\
= \; & E[\pi_I\left(a_0^I = -1, b_0^I = n, a_1^I = -1, b_1^I = n\right) \mid \{-\}] \\
= \; & \lambda\left(\frac{QQ_I}{2} + \frac{QQ_H}{2}\right) + \beta \cdot \Pr\left[\varepsilon_1 > P_1^{**} - \lambda\left(QQ_I + QQ_H\right)\right] \cdot \left(\lambda QQ_M\right) \\
& + \beta^2\left[\widehat{v} - \lambda\left(QQ_I + QQ_H + \Pr\left[\varepsilon_1 > P_1^{**} - \lambda\left(QQ_I + QQ_H\right)\right] \cdot QQ_M\right)\right] \\
= \; & \beta^2\widehat{v} + \lambda\left(\frac{QQ_I}{2} + \frac{QQ_H}{2}\right)\left(1 - 2\beta^2\right) + \lambda\beta QQ_M\left(1 - \beta\right)\left[1 - \Phi\left(\frac{P_1^{**} - \lambda\left(QQ_I + QQ_H\right)}{\sigma_\varepsilon}\right)\right]
\end{aligned}$$

$$\begin{aligned}
& E[\pi_I\left(a_0^I = 1, b_0^I = l, a_1^I = 1, b_1^I = n\right) \mid \{+\}] \\
= \; & E[\pi_I\left(a_0^I = -1, b_0^I = s, a_1^I = -1, b_1^I = n\right) \mid \{+\}] \\
= \; & \lambda\left(\frac{QQ_I}{2} + \frac{QQ_H}{2}\right) + \beta\lambda Q_M + \beta\lambda \cdot \left(QQ_M - Q_M\right) \cdot \Pr\left[\varepsilon_1 > P_1^{**} - \lambda\left(QQ_I + QQ_H\right)\right] \\
& + \beta^2\left[\widehat{v} - \lambda\left(QQ_I + QQ_H + Q_M + \Pr\left[\varepsilon_1 > P_1^{**} - \lambda\left(QQ_I + QQ_H\right)\right]\left(QQ_M - Q_M\right)\right)\right] \\
= \; & \beta^2\widehat{v} + \lambda\left(\frac{QQ_I}{2} + \frac{QQ_H}{2}\right)\left(1 - 2\beta^2\right) \\
& + \left\{\lambda\beta Q_M\left(1 - \beta\right) + \lambda\beta\left(QQ_M - Q_M\right)\left(1 - \beta\right)\left[1 - \Phi\left(\frac{P_1^{**} - \lambda\left(QQ_I + QQ_H\right)}{\sigma_\varepsilon}\right)\right]\right\}
\end{aligned}$$

Since $\beta < 1$, we always have

$$\begin{aligned}
& E[\pi_I\left(a_0^I = 1, b_0^I = l, a_1^I = 1, b_1^I = n\right) \mid \{+\}] = E[\pi_I\left(a_0^I = -1, b_0^I = s, a_1^I = -1, b_1^I = n\right) \mid \{+\}] \mid \{-\}] \\
& > E[\pi_I\left(a_0^I = 1, b_0^I = n, a_1^I = 1, b_1^I = n\right) \mid \{+\}] = E[\pi_I\left(a_0^I = -1, b_0^I = n, a_1^I = -1, b_1^I = n\right)
\end{aligned}$$

Thus, we can conclude informed traders S_I always post truthfully in state ω^- and ω^+ after building their positions because of the time discount factor.

And informed traders also better off in state ω^0 given S_H's strategy:

$$E[\pi_I\left(a_0^I = 0, b_0^I = n; a_1^I = 1\right) \mid \{0\}, b_0^H = s, P_1 < v_0 + P_1^{**}]$$

$$
\begin{aligned}
&= E[\pi_I\left(a_0^I = 0, b_0^I = n; a_1^I = -1\right) | \{0\}, b_0^H = l, P_1 > v_0 - P_1^{**}] \\
&= \Pr\left[P_1 > v_0 + P_1^*\right] \cdot \left\{\lambda\beta\left(QQ_H - Q_H + Q_M - \frac{QQ_I + Q_H}{2}\right)\right\} \\
&\quad + \Pr\left[v_0 < P_1 < v_0 + P_1^*\right] \cdot \left\{\lambda\beta\left(\frac{QQ_H + Q_M}{2} \cdot \frac{QQ_I}{QQ_I + QQ_H}\right)\right\} \\
&\quad + \Pr\left[v_0 - P_1^{**} < P_1 < v_0\right] \cdot \left\{\lambda\beta\left(\frac{QQ_H + Q_M}{2} \cdot \frac{Q_I}{Q_I + QQ_H}\right)\right\} \\
&= \Phi\left(\frac{P_1^* - \lambda QQ_H}{\sigma_\varepsilon}\right) \cdot \left\{\lambda\beta\left(QQ_H - Q_H + Q_M - \frac{QQ_I + Q_H}{2}\right)\right\} \\
&\quad + \left[\Phi\left(\frac{-\lambda QQ_H}{\sigma_\varepsilon}\right) - \Phi\left(\frac{P_1^* - \lambda QQ_H}{\sigma_\varepsilon}\right)\right] \cdot \left\{\lambda\beta\left(\frac{QQ_H + Q_M}{2} \cdot \frac{QQ_I}{QQ_I + QQ_H}\right)\right\} \\
&\quad + \left[\Phi\left(\frac{-P_1^{**} - \lambda QQ_H}{\sigma_\varepsilon}\right) - \Phi\left(\frac{-\lambda QQ_H}{\sigma_\varepsilon}\right)\right] \cdot \left\{\lambda\beta\left(\frac{QQ_H + Q_M}{2} \cdot \frac{Q_I}{Q_I + QQ_H}\right)\right\}
\end{aligned}
$$

Without the communication group,

$$
\begin{aligned}
&\Pr\left\{S_H = \{+, 0\} | P_1, \omega^0\right\} \\
= \;& \frac{\phi\left(\frac{P_1 - v_0 - \lambda QQ_H}{\sigma_\varepsilon}\right) \cdot \frac{1}{2} \cdot (1 - 2p)}{\phi\left(\frac{P_1 - v_0 - \lambda QQ_H}{\sigma_\varepsilon}\right) \cdot \frac{1}{2} \cdot (1 - 2p) + \phi\left(\frac{P_1 - v_0 + \lambda QQ_H}{\sigma_\varepsilon}\right) \cdot \frac{1}{2} \cdot (1 - 2p)} \\
\triangleq \;& p^{**}
\end{aligned}
$$

$$
\begin{aligned}
& E[\pi_I\left(a_0^I = 0, a_1^I = -1\right) | \{0\}, P_1 > v_0] \\
= \;& E[\pi_I\left(a_0^I = 0, a_1^I = 1\right) | \{0\}, P_1 < v_0] \\
= \;& p^{**}\left\{\Pr\left[v_0 < P_1 < v_0 + P_1^*\right] \cdot \lambda\beta\left(\frac{QQ_H}{2} \cdot \frac{QQ_I}{QQ_I + QQ_H}\right)\right. \\
& \left. + \Pr\left[P_1 > v_0 + P_1^*\right] \cdot \lambda\beta\left(QQ_H - \frac{QQ_I}{2}\right)\right\} \\
= \;& p^{**}\left\{\left[\Phi\left(\frac{P_1^* - \lambda QQ_H}{\sigma_\varepsilon}\right) - \Phi\left(\frac{-\lambda QQ_H}{\sigma_\varepsilon}\right)\right] \cdot \lambda\beta\left(\frac{QQ_H}{2} \cdot \frac{QQ_I}{QQ_I + QQ_H}\right)\right. \\
& \left. + \Phi\left(\frac{P_1^* - \lambda QQ_H}{\sigma_\varepsilon}\right) \cdot \lambda\beta\left(QQ_H - \frac{QQ_I}{2}\right)\right\}
\end{aligned}
$$

Obviously,

$$
E[\pi_I\left(a_0^I = 0, b_0^I = n; a_1^I = -1\right) | \{0\}, b_0^H = l, P_1 > v_0 - P_1^{**}]
$$

$$
\begin{aligned}
&= E[\pi_I\left(a_0^I = 0, b_0^I = n; a_1^I = 1\right) \mid \{0\}, b_0^H = s, P_1 < v_0 + P_1^{**}] \\
&> E[\pi_I\left(a_0^I = 0, a_1^I = -1\right) \mid \{0\}, P_1 > v_0] \\
&= E[\pi_I\left(a_0^I = 0, a_1^I = 1\right) \mid \{0\}, P_1 < v_0]
\end{aligned}
$$

In the cases $\{\{0\}, P_1 < v_0 - P_1^{**}\}$ and $\{\{0\}, P_1 > v_0 + P_1^{**}\}$, inside informed traders take the same action and make the same profits as outside informed traders.

Therefore, informed traders always better off within the group.

For momentum traders S_M,

$$
\begin{aligned}
\Pr[\omega^+|b_0^{-M} &= l] \\
&= \frac{\Pr[l|\omega^+] \cdot \Pr[\omega^+]}{\Pr[l|\omega^+] \cdot \Pr[\omega^+] + \Pr[l|\omega^0] \cdot \Pr[\omega^0] + \Pr[l|\omega^-] \cdot \Pr[\omega^-]}, \\
&= \frac{1 \cdot p}{1 \cdot p + \frac{1}{2} \cdot (1 - 2p) + 0} \\
&= 2p
\end{aligned}
$$

$$
\begin{aligned}
&\Pr[\omega^+|b_0^{-M} = l, P_1] \\
= &\frac{\Pr[P_1|\omega^+] \cdot \Pr[\omega^+|l]}{\Pr[P_1|\omega^+] \cdot \Pr[\omega^+|l] + \Pr[P_1|\omega^0] \cdot \Pr[\omega^0|l] + \Pr[P_1|\omega^-] \cdot \Pr[\omega^-|l]} \\
= &\frac{\Pr[P_1|\omega^+] \cdot 2p}{\Pr[P_1|\omega^+] \cdot 2p + \Pr[P_1|\omega^0] \cdot (1 - 2p)} \\
= &\frac{\Pr[\varepsilon = P_1 - v_0 - \lambda\left(QQ_I + QQ_H\right)|\omega^+] \cdot 2p}{\Pr[\varepsilon = P_1 - v_0 - \lambda\left(QQ_I + QQ_H\right)|\omega^+] \cdot 2p + \Pr[\varepsilon = P_1 - v_0|\omega^0] \cdot (1 - 2p)} \\
= &\frac{\phi\left(\frac{P_1 - v_0 - \lambda(QQ_I + QQ_H)}{\sigma_\varepsilon}\right) \cdot 2p}{\phi\left(\frac{P_1 - v_0 - \lambda(QQ_I + QQ_H)}{\sigma_\varepsilon}\right) \cdot 2p + \phi\left(\frac{P_1 - v_0}{\sigma_\varepsilon}\right) \cdot (1 - 2p)} \\
= &p^*
\end{aligned}
$$

and $\Pr[\omega^0|b_0^{-M} = l, P_1] = 1 - p^*$, $\Pr[\omega^-|b_0^{-M} = l, P_1] = 0$.

Thus, the posts in the group help momentum traders exclude one state. With others' posts, momentum traders attain the same informative signal as outside hybrid traders.

$$
\begin{aligned}
& E[\pi_M\left(a_0^M=0, a_1^M=1\right)|b_1^{-M}=l, P_1] \\
= \;& p^*\left\{0+\beta\lambda\left(\frac{Q_M}{2}+\Pr\left[P_1>v_0+P_1^{**}\right]\cdot\frac{QQ_M-Q_M}{2}\right)\right. \\
& \left.+\beta^2\left[\widehat{v}-\lambda\left(QQ_I+QQ_H+Q_M+\Pr\left[P_1>v_0+P_1^{**}\right]\cdot\left(QQ_M-Q_M\right)\right)\right]\right\} \\
& +(1-p^*)\left\{0+\beta\lambda\left(\frac{Q_M}{2}+\Pr\left[P_1>v_0+P_1^{**}\right]\cdot\frac{QQ_M-Q_M}{2}\right)\right. \\
& \left.+\beta^2\lambda-Q_M-\Pr\left[P_1>v_0+P_1^{**}\right]\cdot\left(QQ_M-Q_M\right)\right\} \\
= \;& \lambda\beta\frac{QQ_M}{2}(1-2\beta)+\beta\lambda\frac{QQ_M-Q_M}{2}(1-2\beta)\left[1-\Phi\left(\frac{P_1^*-\lambda QQ_H}{\sigma_\varepsilon}\right)\right] \\
& +p^*\left\{\beta^2\left[\widehat{v}-\lambda\left(QQ_I+QQ_H\right)\right]\right. \\
& \left.+\beta\lambda\frac{QQ_M-Q_M}{2}(1-2\beta)\left[\Phi\left(\frac{P_1^*-\lambda QQ_H}{\sigma_\varepsilon}\right)-\Phi\left(\frac{P_1^*-\lambda\left(QQ_I+QQ_H\right)}{\sigma_\varepsilon}\right)\right]\right\}
\end{aligned}
$$

$$
\begin{aligned}
& E[\pi_M\left(a_0^M=0;a_1^M=1\right)|b_1^{-M}=l, P_1]>E[\pi_M\left(a_0^M=0;a_1^M=0\right)|l, P_1]=0 \\
\Longrightarrow p^* > \;& \frac{\lambda\beta\frac{QQ_M}{2}(1-2\beta)+\beta\lambda\frac{QQ_M-Q_M}{2}(1-2\beta)\left[1-\Phi\left(\frac{P_1^*-\lambda QQ_H}{\sigma_\varepsilon}\right)\right]}{\beta^2\left[\widehat{v}-\lambda\left(QQ_I+QQ_H\right)\right]+\beta\lambda\frac{QQ_M-Q_M}{2}(1-2\beta)\left[\Phi\left(\frac{P_1^*-\lambda QQ_H}{\sigma_\varepsilon}\right)-\Phi\left(\frac{P_1^*-\lambda(QQ_I+QQ_H)}{\sigma_\varepsilon}\right)\right]}
\end{aligned}
$$

Denote $\frac{\lambda(1-2\beta)\left\{\beta\frac{QQ_M}{2}+\beta\frac{QQ_M-Q_M}{2}\left[1-\Phi\left(\frac{P_1^*-\lambda QQ_H}{\sigma_\varepsilon}\right)\right]\right\}}{\beta^2[\widehat{v}-\lambda(QQ_I+QQ_H)]+\beta\lambda\frac{QQ_M-Q_M}{2}(1-2\beta)\left[\Phi\left(\frac{P_1^*-\lambda QQ_H}{\sigma_\varepsilon}\right)-\Phi\left(\frac{P_1^*-\lambda\left(QQ_I+QQ_H\right)}{\sigma_\varepsilon}\right)\right]} \triangleq K_2.$

$$
\begin{aligned}
E[\pi_M\left(a_0^M=0;a_1^M=1\right)|b_1^{-M} &= l, P_1]>E[\pi_M\left(a_0^M=0;a_1^M=0\right)|l, P_1]=0 \\
&\Longrightarrow P_1>v_0+P_1^{***} \\
\text{where } P_1^{***} &= \lambda\left(QQ_H+\frac{QQ_I}{2}\right)-\frac{\sigma_\varepsilon^2}{\lambda QQ_I}\cdot\ln\left(\frac{2p\left(1-K_2\right)}{K_2\left(1-2p\right)}\right)
\end{aligned}
$$

When $\frac{2p(1-K_2)}{K_2(1-2p)}\cdot\frac{1+K_1}{1-K_1}>e^{\frac{\lambda^2\left(QQ_I+QQ_H\right)^2}{\sigma_\varepsilon^2}}$, we have $P_1^{**}>P_1^{***}$. Given that noise σ_ε in the price cannot be ignored in S_I and S_H's price impact $\lambda\left(QQ_I+QQ_H\right)$, since momentum traders inside the chatroom have more information than those outside the chatroom, they do not need the price path walk so far to confirm their estimate and they can enter the market with a lower threshold.

In short, inside momentum traders are better off within the group because of more information.

For hybrid traders,

$$
\begin{aligned}
& E\left[\pi_H\left(a_0^H=1, b_0^H=l ; a_1^H=1\right) \mid\{+, 0\}, b_0^I=l\right] \\
= & E\left[\pi_H\left(a_0^H=-1, b_0^H=s ; a_1^H=-1\right) \mid\{-, 0\}, b_0^I=s\right] \\
= & \lambda \frac{QQ_I+QQ_H}{2}+\beta \lambda Q_M+\beta \lambda \cdot\left(QQ_M-Q_M\right) \cdot \operatorname{Pr}\left[\varepsilon>P_1^{**}-\lambda\left(QQ_I+QQ_H\right)\right] \\
& +\beta^2\left[\widehat{v}-\lambda\left(QQ_I+QQ_H+Q_M+\operatorname{Pr}\left[\varepsilon>P_1^{**}-\lambda\left(QQ_I+QQ_H\right)\right]\left(QQ_M-Q_M\right)\right)\right] \\
= & \beta^2 \widehat{v}+\lambda\left(\frac{QQ_I}{2}+\frac{QQ_H}{2}\right)\left(1-2 \beta^2\right)+\lambda \beta QQ_M(1-\beta)\left[1-\Phi\left(\frac{P_1^{**}-\lambda\left(QQ_I+QQ_H\right)}{\sigma_{\varepsilon}}\right)\right]
\end{aligned}
$$

$$
\begin{aligned}
& E\left[\pi_H\left(a_0^H=1, b_0^H=l ; a_1^H=-1\right) \mid\{+, 0\}, b_1^I=n, P_1>v_0-P_1^{**}\right] \\
= & E\left[\pi_H\left(a_0^H=-1, b_0^H=s ; a_1^H=1\right) \mid\{-, 0\}, b_0^I=n, P_1<v_0+P_1^{**}\right] \\
= & \operatorname{Pr}\left[P_1>v_0+P_1^*\right] \cdot\left\{\lambda \beta\left(QQ_H-Q_H+Q_M-\frac{QQ_I+Q_H}{2}\right)\right\} \\
& +\operatorname{Pr}\left[v_0<P_1<v_0+P_1^*\right] \cdot\left\{\lambda \beta\left(\frac{QQ_H+Q_M}{2} \cdot \frac{QQ_I}{QQ_I+QQ_H}\right)\right\} \\
& +\operatorname{Pr}\left[v_0-P_1^{**}<P_1<v_0\right] \cdot\left\{\lambda \beta\left(\frac{QQ_H+Q_M}{2} \cdot \frac{Q_I}{Q_I+QQ_H}\right)\right\} \\
= & \Phi\left(\frac{P_1^*-\lambda QQ_H}{\sigma_{\varepsilon}}\right) \cdot\left\{\lambda \beta\left(QQ_H-Q_H+Q_M-\frac{QQ_I+Q_H}{2}\right)\right\} \\
& +\left[\Phi\left(\frac{-\lambda QQ_H}{\sigma_{\varepsilon}}\right)-\Phi\left(\frac{P_1^*-\lambda QQ_H}{\sigma_{\varepsilon}}\right)\right] \cdot\left\{\lambda \beta\left(\frac{QQ_H+Q_M}{2} \cdot \frac{QQ_I}{QQ_I+QQ_H}\right)\right\} \\
& +\left[\Phi\left(\frac{-P_1^{**}-\lambda QQ_H}{\sigma_{\varepsilon}}\right)-\Phi\left(\frac{-\lambda QQ_H}{\sigma_{\varepsilon}}\right)\right] \cdot\left\{\lambda \beta\left(\frac{QQ_H+Q_M}{2} \cdot \frac{Q_I}{Q_I+QQ_H}\right)\right\}
\end{aligned}
$$

In the two cases $\left\{\{+,0\}, b_1^I=n, P_1>v_0-P_1^{**}\right\}$ and $\left\{\{-,0\}, b_1^I=n, P_1>v_0+P_1^{**}\right\}$, insider hybrid traders attain the same payoffs as outside hybrid traders.

Obviously, in all the three states, inside hybrid traders are better off within the group.

Table 1
Signals and States

Trader i	Signal θ_i at $t=0$		
	state ω^+ $\widetilde{v}=v_0+\widehat{v}$	state ω^- $\widetilde{v}=v_0-\widehat{v}$	state ω^0 $\widetilde{v}=v_0$
S_I	$\{+\}$	$\{-\}$	$\{0\}$
S_H	$\{0,+\}$	$\{0,-\}$	$\begin{cases} \{0,+\} \text{ with prob } \frac{1}{2} \\ \{0,-\} \text{ with prob } \frac{1}{2} \end{cases}$
S_M	$\{+,0,-\}$	$\{+,0,-\}$	$\{+,0,-\}$

Table 2
Summary of Posts and Trades

Year:	2000
Number of posts	77,712
Trades	3,658
Number of Posters	2,184
Overall Profits	$349,578.10
Profit Per Trade	$135.06
% Profitable	52.82%

Table 3
Empirical Tests

Hypothesis	Dep. Var.	π_j	π_j^-	π_j^+	R^2
H1 A	TRP_j		11.624 (2.61)		6.2%
H1 B	TRP_j			-15.371 (-2.02)	3.1%
H2a	F_j	-0.974 (-2.27)			9.2%
H2b	$\pi_j^{(f)} - \pi_j^{(nf)}$	-2.152 (-5.60)			38.1%
H3	F_{-j}	0.062 (2.73)			17.9%

In H1, we limit the sample to traders with more than 1 trade and more than 10 posts and exclude 6 traders on the tails of profit/trade, including high&mid skill level traders whose profits are between -4 and 0.1 in H1A and low&mid skill level traders whose profits between -0.1 and 4 in H1B. In H2, we include only those who follow more than 1 time. In H3, we limit the sample to traders who are followed by others more than once but not always been followed.

Figure 1

Price Path: State ω^{+}

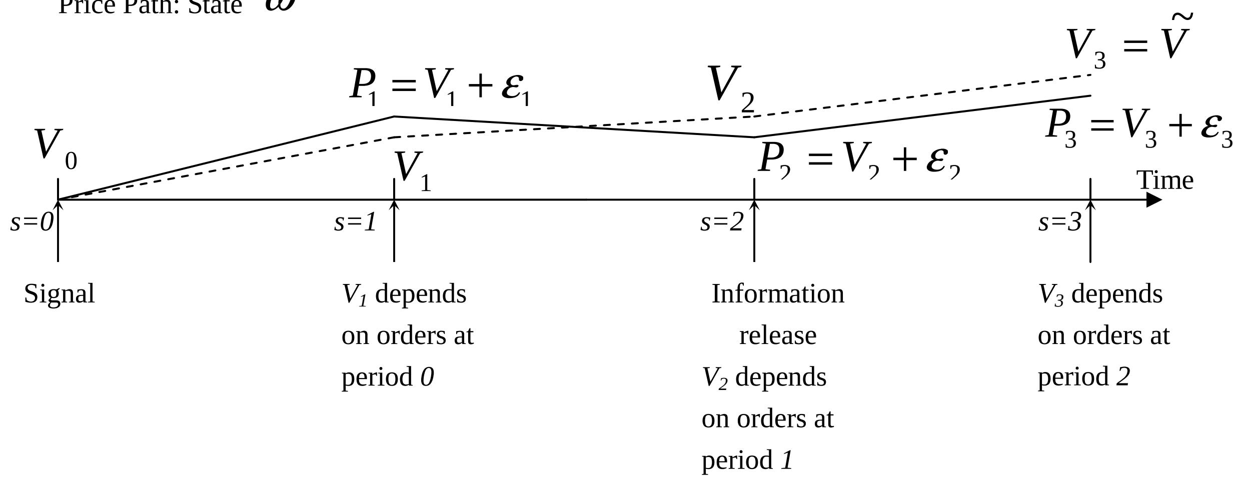

Figure 2

State ω^{+} , $S_I : \{+\}$

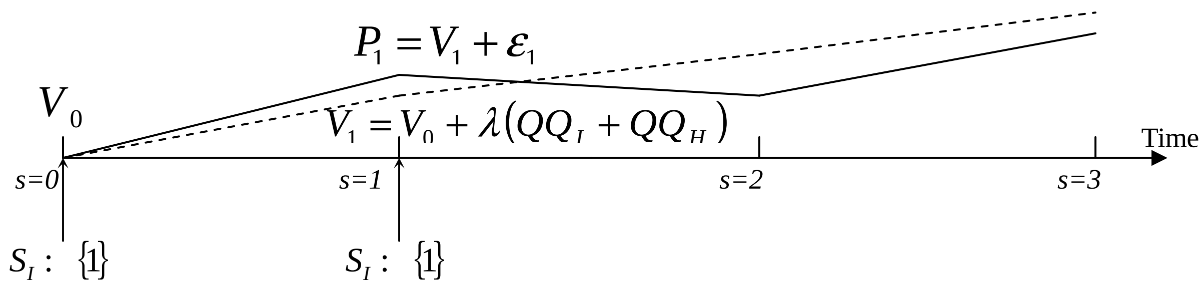

Figure 3

State ω^{+} , $S_H : \{+,0\}$

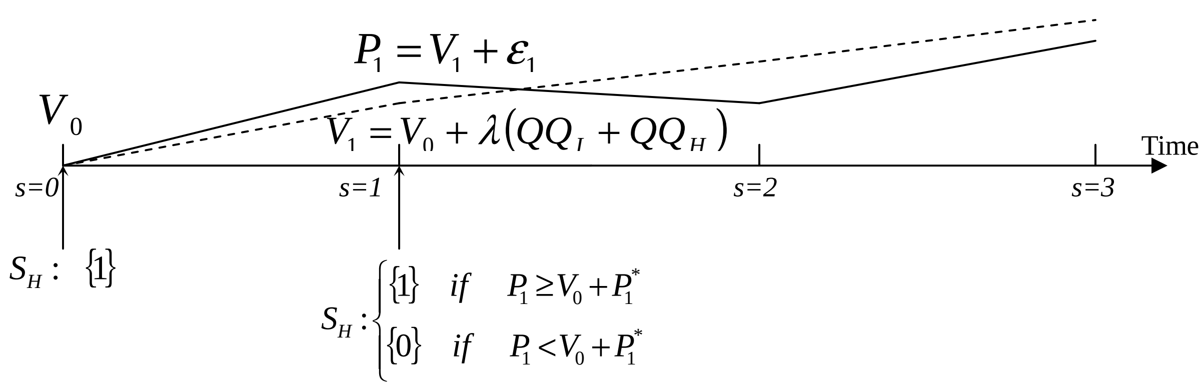

Figure 4

State ω^{+} , $S_M : \{+,0,-\}$

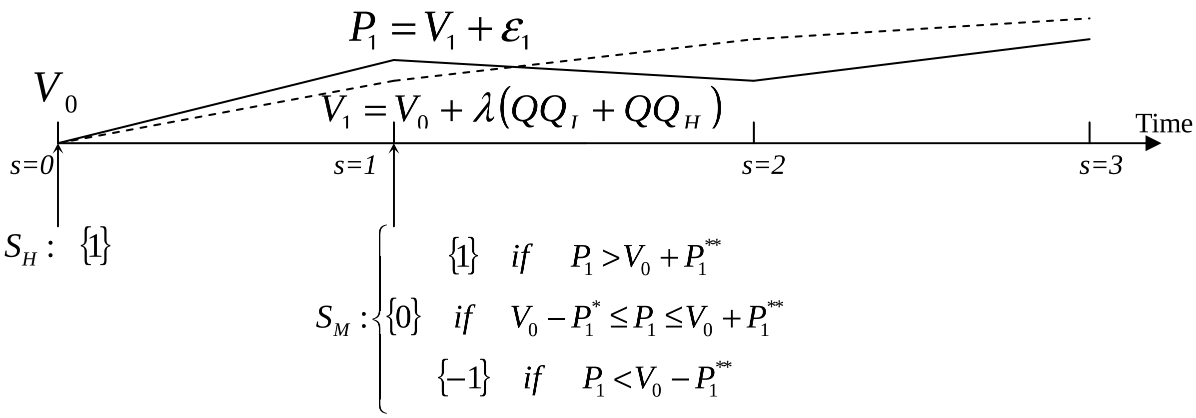

Figure 5

$S_I : \{+\}$

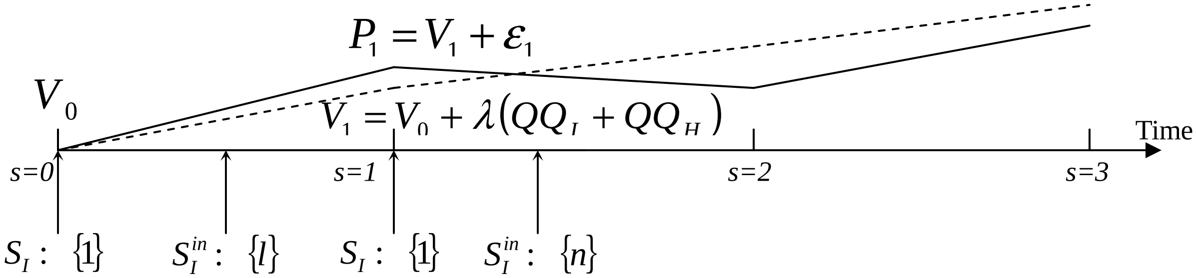

Figure 6

$S_I : \{0\} \quad with \quad S_H : \{l\}$

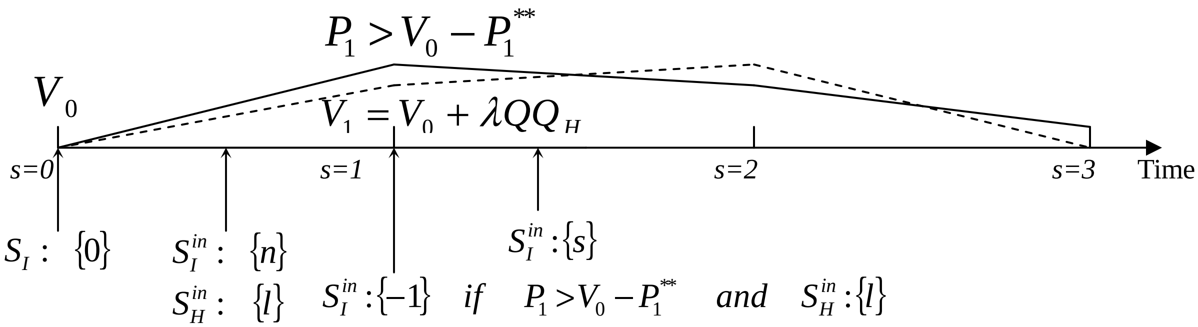

Figure 7

$S_H : \{+,0\} \quad with \quad S_I : \{l\}$

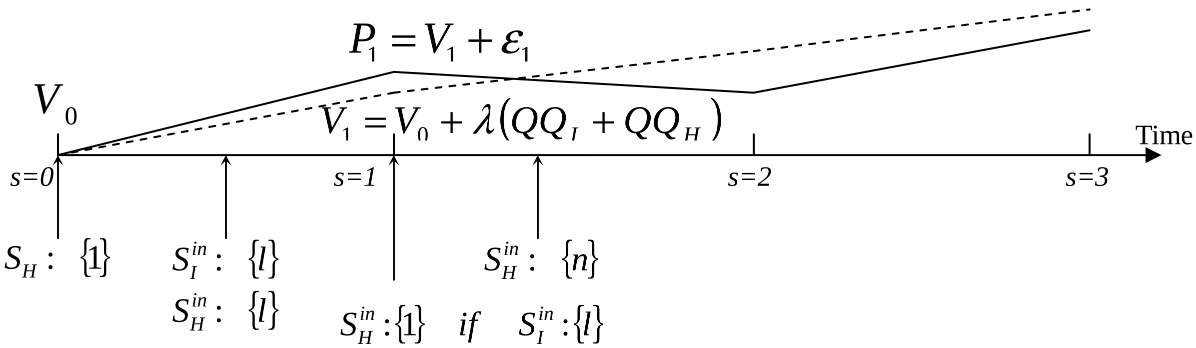

Figure 8

$S_H : \{+,0\} \quad with \quad S_I : \{n\}$

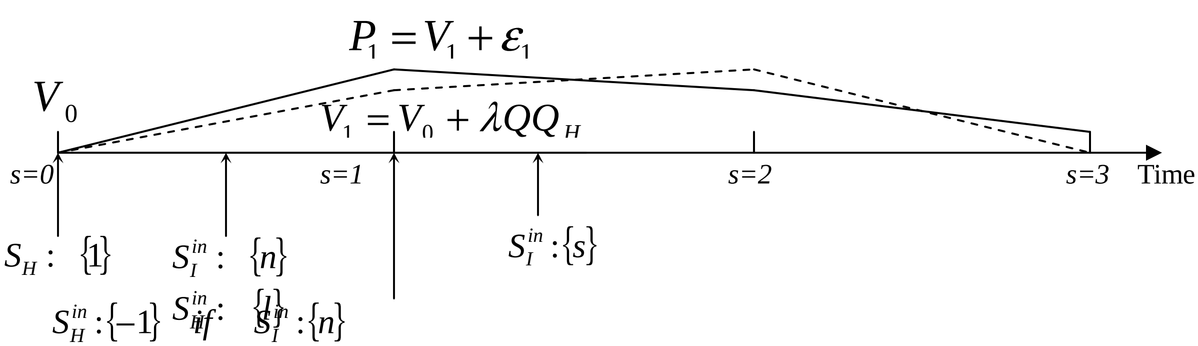

Figure 9

$S_M : \{+,0,-\} \quad with \quad S_{-M} : \{l\}$